

Today

- Test Prep – first 10 minutes

- Office Hours Today:

3:00-4:30pm, Com B-006

- HW 1 closes Mon (1.1-1.2)

HW 1 overview

1, 3: Algebra review

2: Integration review

4-6: Check a sol'n

7-8: Applications & units

9-10: Applications set up

- Lecture

a) More motivating examples

b) Slope/Direction Fields

→ How are you doing on these?

$$y(x) = e^x + rx + sx^2$$

$$y'' - y' = 3x$$

WANT TO SEE?

Ch. 1: Motivating Examples

Recall: $\frac{dy}{dx} = \text{rate}$

Rates come up everywhere!

$$\frac{dT}{dt} = -0.5(T-70) \quad \frac{^{\circ}\text{F}}{\text{minute}}$$

Newton's Law of cooling

- a) Populations, Savings Accounts,
Newton's Law of cooling,
Mixing Problems, melting ice
(see handout)

$$\frac{dP}{dt} = 0.03P \quad \frac{\text{people}}{\text{yr}}$$

3% relative growth rate

$$\frac{dA}{dt} = 0.02A \quad \frac{\text{dollars}}{\text{yr}}$$

2% interest rate
(compounded continuously)

$$\frac{dA}{dt} = 0.02A + 100 \quad \frac{\text{dollars}}{\text{yr}}$$

2% interest rate
plus 100 deposited
per year

b) Free-fall (no air resistance):

$$mv' = F_g = -mg$$

Initial Value Problem (IVP)

$$v' = -g$$

$$v(0) = 0 \quad \leftarrow \text{DROPPED}$$

Newton's Law

$$\text{Force} = ma = m v' = m \dot{v} = m \frac{dv}{dt}$$

ASIDE: 3 ways you'll see this

Force due to gravity = mg (downward)

So $ma = \text{Force} = -mg$

$$m \frac{dv}{dt} = -mg$$

$$\boxed{\frac{dv}{dt} = -9.8}$$

$$\Rightarrow \boxed{v(t) = -9.8t + C}$$

c) ...with air resistance

$$mv' = F_g + F_A = -mg - rv$$

$$v' = -g - \frac{r}{m}v$$

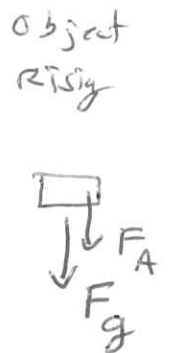
$$v(0) = 0$$

ONE MODEL

↳ "Air resistance proportional to velocity"

r = proportionality constant

$$\frac{dv}{dt} = -g - \frac{r}{m}v \quad \text{CAN'T INTEGRATE!!}$$



d) Mass-Spring Example:

$$\text{Force} = -kx$$

$$m x'' = -kx$$

It turns out that one solution to this is $x(t) = \cos(\omega t)$

$$x' = -\omega \sin(\omega t)$$

$$x'' = -\omega^2 \cos(\omega t)$$

$$-m \omega^2 \cos(\omega t) \stackrel{?}{=} -k \cos(\omega t)$$

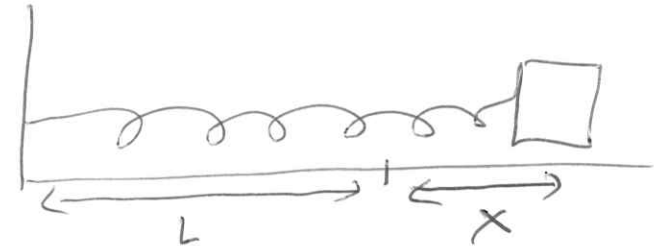
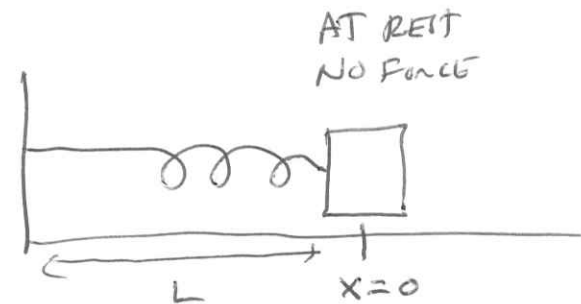
↑ NEED TO BE SAME! ↑

$$m \omega^2 = k$$

$$\Rightarrow \omega^2 = \frac{k}{m}$$

$$\omega = \pm \sqrt{\frac{k}{m}}$$

k = force to hold mass "one unit" away from natural length



$$x(t) = \cos\left(\sqrt{\frac{k}{m}} t\right)$$

SIMPLE HARMONIC OSCILLATION

$$\sqrt{\frac{k}{m}} = \text{natural frequency} \quad \frac{\text{radians}}{\text{sec}}$$

$$\frac{2\pi}{\omega} = \text{wavelength}$$

e) Circuits (read lecture notes, ← From my WEBSITE
follow instructions on HW):

$$V = Rq' + \frac{1}{C}q + Lq''$$

IN the problem, $V_L = 0$ ($L=0$) \Rightarrow $V = R \frac{dq}{dt} + \frac{1}{C}q$

part (a)

$$\frac{dq}{dt} = ??$$

part (b)

$$V_C(t) = \frac{q(t)}{C}$$

$$\Rightarrow \frac{dV_C}{dt} = \frac{1}{C} \frac{dq}{dt}$$

USE THIS

Slope/Direction Fields

Recall: $\frac{dy}{dx} = \text{slope}$

We can visualize slope!

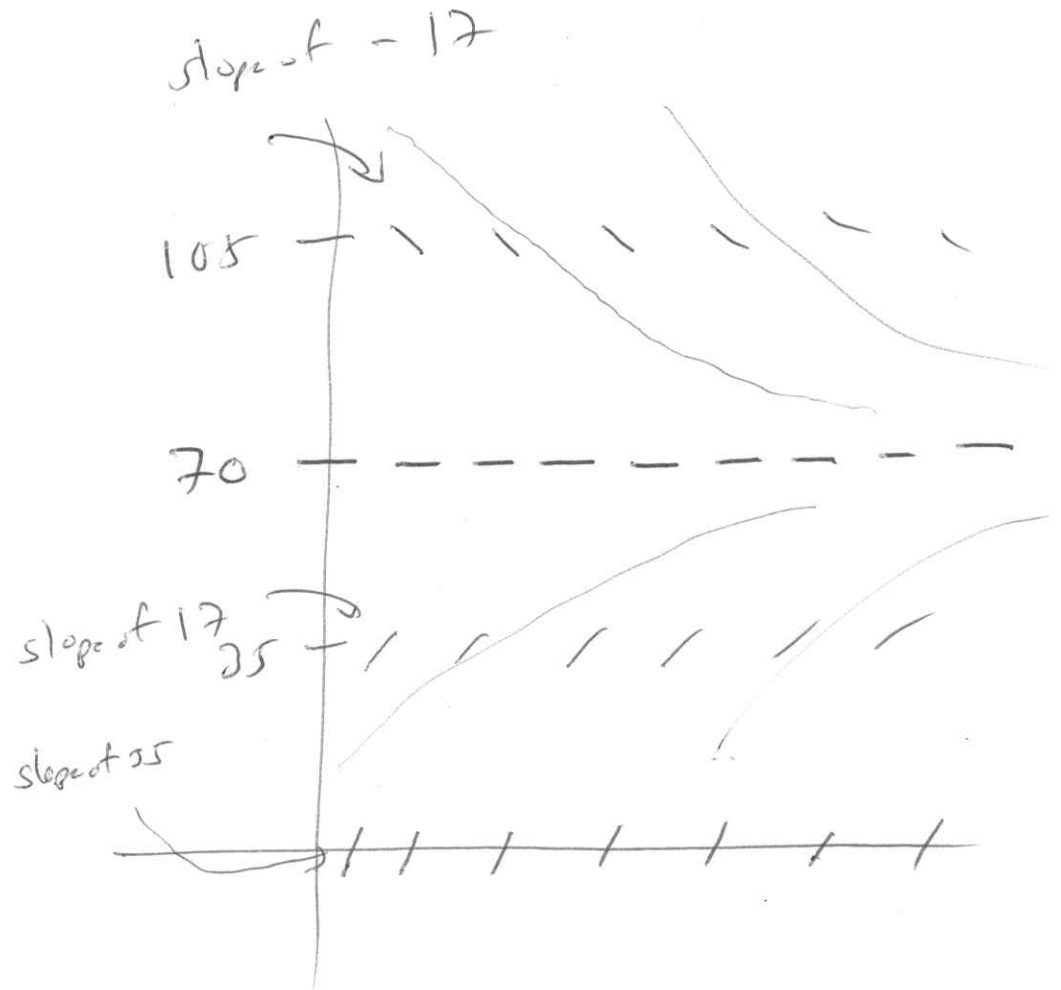
Example 1:

$$\frac{dT}{dt} = 0.5(70 - T)$$

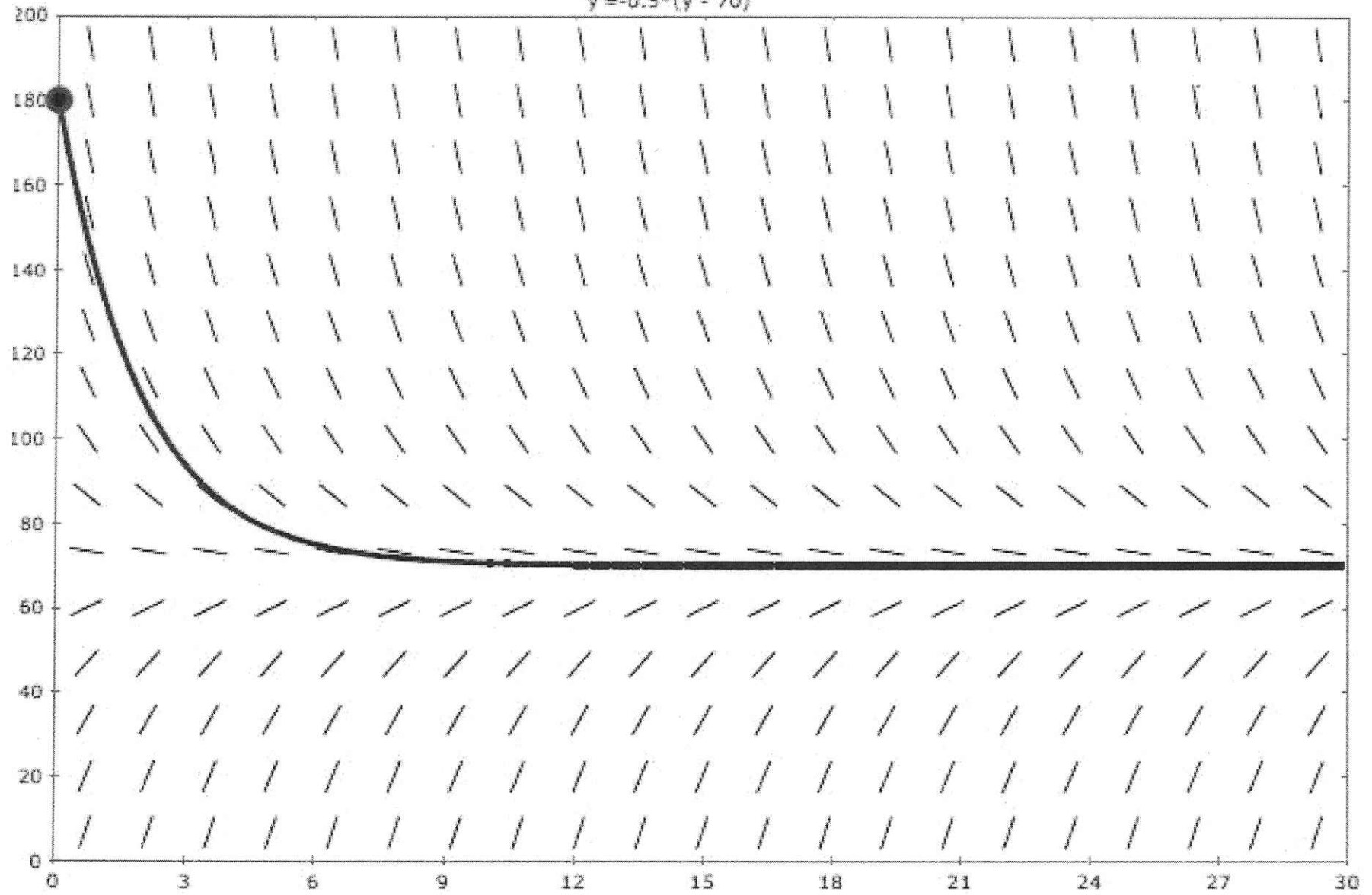
	T=0	T=35	T=70	T=105
t=0	35	17	0	-17
t=10	35	17	0	-17
t=20	35	17	0	-17
t=30	35	17	0	-17

↑

autonomous (doesn't depend on t)



$$y' = -0.5 \cdot (y - 70)$$



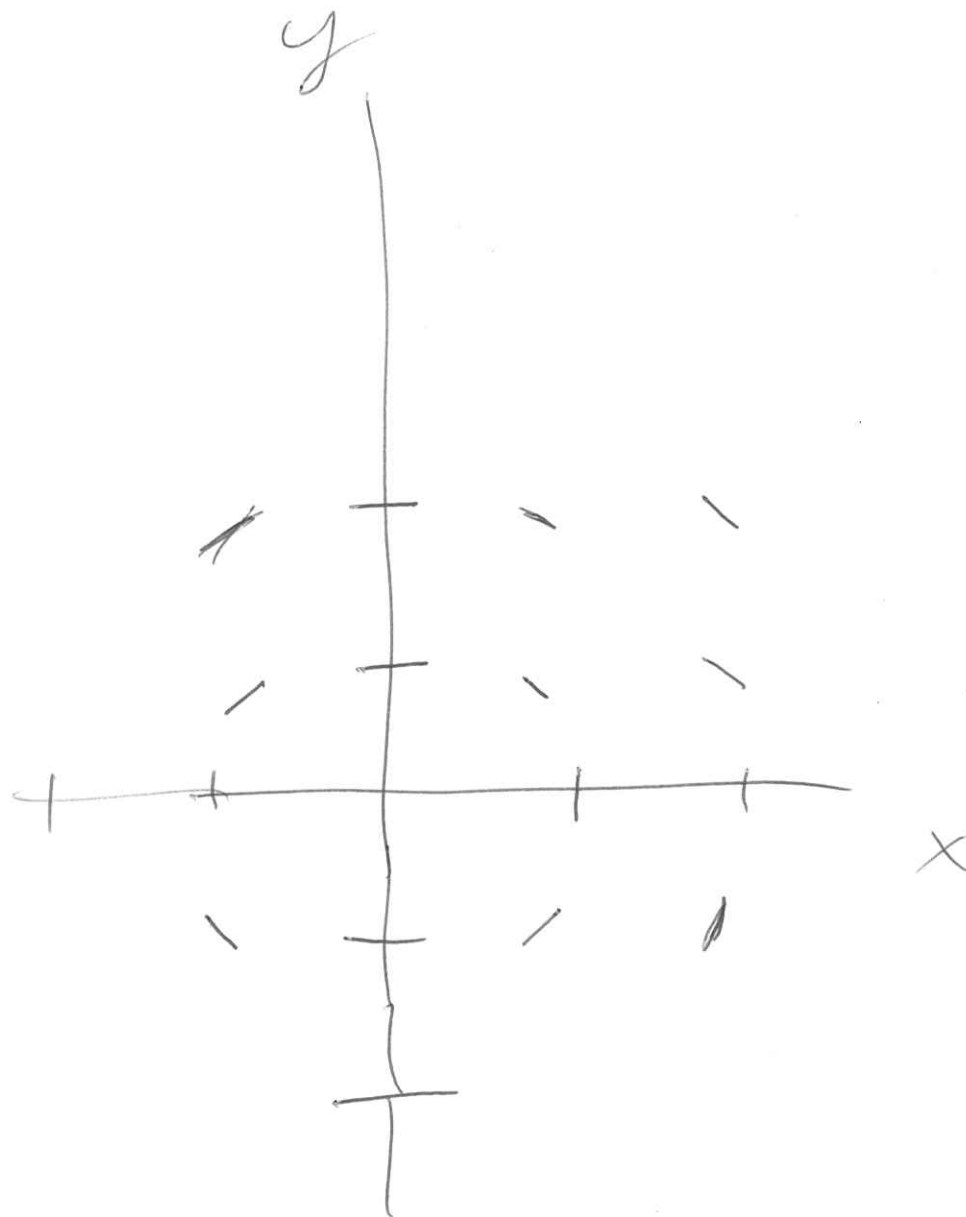
Example 2:

depends on both
 x & y

$$\frac{dy}{dx} = -\frac{x}{y}$$

	$y=-1$	$y=0$	$y=1$	$y=2$
$x=-1$	-1	?	1	$\frac{1}{2}$
$x=0$	0	0	0	0
$x=1$	+1	?	-1	$-\frac{1}{2}$
$x=2$	2	?	-2	-1

vertical
tangent



$$y' = -t/y$$

